NON-GAUSSIAN BEHAVIOR OF SOME COMMODITIES PRICES EMPIRICAL EVIDENCES IN BRAZILIAN MARKETS

José Augusto M. de Andrade Júnior
Faculdade de Economia Administração e Contabilidade de Ribeirão Preto – FEA-RP/USP
Departamento de Administração
Av. Bandeirantes, 3900 Ribeirão Preto – SP –Brasil CEP: 14040-900
E-mail: **jamaj@jamaj.com.br**

Tabajara Pimenta Júnior
Universidade de São Paulo - USP
Faculdade de Economia Administração e Contabilidade de Ribeirão Preto – FEA-RP/USP
Departamento de Administração
Av. Bandeirantes, 3900 Ribeirão Preto – SP –Brasil CEP: 14040-900
E-mail: taba.jr@terra.com.br

Abstract

Since 1960´s, when Mandelbrot showed that price time-series on some speculative markets have some characteristics that diverge from the normality implied by random walk model, many authors around the world have found evidences of what are now known as stylized facts of these markets: the non-Gaussian behavior, the volatility clustering, the long-term memory and "heavy or long tail" of price returns distribution. In this work the impact of non-normality on the finance engineering tools is showed. Next, evidences of this statistical stylized facts on some Brazilian commodity markets are searched for, by using non-parametric normality tests to verify or reject the null Gaussian hypothesis and for independence (i.i.d.). The results found allow the rejection of the null hypothesis of normality and for i.i.d.

Key words: *non-parametric tests, speculative markets,commodities, time-series analysis, non-Gaussian hypothesis, heavy tails*

NON-GAUSSIAN BEHAVIOR OF SOME COMMODITIES PRICES EMPIRICAL EVIDENCES IN BRAZILIAN MARKETS

1. Introduction

Since Mandelbrot´s seminal paper "The Variation of Certain Speculative Prices" (Mandelbrot(1963)), many evidences have been found by many researchers around the world, that reject the normality assumption of price variation in speculative markets (see Sun(2007), Peters(1994), Hyung(2007)). This non-normal behavior has impact on options pricing, risk modeling and management and markets regulation. This impacts are showed and discussed later with the results.

The main purpose of this work is to search for these stylized facts and to test the Gaussian hypothesis on the variation of prices of some commodities in Brazilian market and to discuss the impacts of these abnormal behavior to finance engineering techniques.

Non-parametric normality tests are applied on Alcohol, Sugar, Cotton, Coffee and SoyBean price time-series given by CEPEA(2007) / ESALQ / USP and the results are then discussed.

2. The financial prices "stylized facts"

This non-Gaussian behavior of price variation account for the appearance of some typical characteristics normally called "stylized facts":

- high kurtosis – "long" or "heavy" tails of distribution
- volatility clustering
- long memory
- self-similarity

Maldelbrot and others (Mandelbrot(1963), Mandelbrot(1963b), Mandelbrot(1997), Mandelbrot & Hudson(2004), Sun(2007)) observed financial asset prices have a distribution that are far from Gaussian normal, especially in the tails of distributions. In other words, the variations of prices are more frequent and greater than expected if they were really normal. These empirical distributions have a kurtosis greater than 3, the normal kurtosis.

Volatility, defined as absolute variation of prices, tend to be "clustered". It means that the current prices are dependent on past prices. This is sometimes called conditional heteroskedasticity, usually modeled by GARCH (generalized autoregressive conditional heteroskedasticity).

The long memory fact means that current prices depends on past prices. This violates the independence assumption. If current prices are not independent, they obviously are not i.i.d. In consequence, the underlying process cannot be characterized as a Wiener process, which in turn is an important assumption to modern finance engineering

Maldelbrot(1963b) observed financial asset prices behave like fractals, that are self-similar, i.e., at every scale they have some geometrical properties that are invariant. It means that some characteristics of a time-series sampled at 1 min rate are the same of the monthly sampling rate (for more references, see Kim(2004) , Peters(1994), Plerou(2000)).

3. Impacts of these "stylized facts" in modern finance engineering

The Gaussian assumption is pervasive in modern finance engineering. Assets pricing, Options pricing, portfolio selection, risks evaluation, banking and insurance regulation and other issues are commonly done with the usage of tools that implicitly make this assumption,

In the next sections the analisys of the impacts of non-Gaussian behavior are made.

3.1 The Bachelier's Gaussian model for price variations and options pricing

The first options pricing model was due Bachelier's thesis "Théorie de la Spéculation" (see Davis & Etheridge(2006)), defended in Sorbonne on 29 march 1900 . Much ahead his time, as he anticipated Einstein's Brownian motion in five years and Black-Scholes option pricing method in eight decades, this thesis remained forgotten until its rediscovery by Jimmie Savage and Samuelson in the 1950s (see Laskin(2000)).

This thesis was the first time prices and its variations was formally considered random variables that follow the Gaussian (normal) distribution on its increments. Bachelier considered pricing the *rentés*, bonds issued by French Real Treasury to financiate the war (see Davis & Etheridge(2006))

3.2 Options and Assets pricing

The Black-Scholes (BS) Nobel Prize winning formula is intrinsically based on Gaussian assumption. Indeed, BS formula is achieved integrating the stochastic differential equation (SDE):

$$\mathbf{dS(t) / S(t) = r\, dt + \sigma\, dW(t)} \qquad (1)$$

where W(t) is a standard Brownian motion and S(t) is the price time-series.

This SDE implies that the price time-series (S(t)) varies based upon r and σ, the rate of return and volatility, respectively. This means that, in the core of BS formula (which results was in fact discovered by Bachelier, as stated in previous section), the increments on prices follow the Gaussian bell curve (dW(t) term in SDE).

Putting the r (return rate) equal to interest rate of economy, the solution of this SDE (1) then becomes:

$$\mathbf{S(T) = S(0) \exp\left([\, r - 1/2\, \sigma^2]\ T + \sigma W(T) \right)} \qquad (2)$$

where S(0) is the current price of the stock, which is known. The random variable W(T) is normally distributed with mean 0 and variance T; this is also the distribution of $T^{1/2}Z$ if Z is a standard normal random variable (mean 0 and variance 1). The expression (2) can now be written as:

$$\mathbf{S(T) = S(0) \exp\left([\, r - 1/2\, \sigma^2]\ T + \sigma\, T^{1/2} Z \right)} \qquad (3)$$

The logarithm of the stock price is thus normally distributed, and the stock price itself has a log-normal distribution.

3.2 Financial risk modeling

Although the many deficiencies of VAR (Value at Risk) (eg Embrechts et al.(1997), Giesecke(2005)) as a measure of financial risk, it is a standard *de facto* method to calculate financial risks. The Baslee Committee uses it to regulate banking activities, what accounts for the widespread usage of VAR. Some authors consider VAR as a measure of capital adequacy to determine if a bank or insurer has sufficient capital to support great losses in its portfolios.

Given $F_L(x) = P(L < x)$, the distribution of losses, the $VAR_{99\%} = 1 - F_L(x_p) \equiv P(L > x) = p$ with $p = 0.01$.

In other words, the VAR is the expected value of losses of a portfolio; it depends upon the distribution of the prices of its assets

The problem is that the expected values of losses normally are calculated assuming the prices will be stationary Gaussian distributed, which means that the parameters of normal (variance and mean) will be always the same. Recent market crashes show that the behavior of portfolios are far different and more wild than expected under normal assumptions. In these situations, volatility are much higher than in other situations. This means that prices variations of assets are sometimes dependents of the others; they are not i.i.d. (identically independently distributed).

If the prices variation of assets in a portfolio were normally distributed (Wiener process or Brownian motion in BS formula), their sum would have a normal distribution, since the sum of i.i.d. random variables are asymptotically normal, cause the Gaussian distribution is Lèvy- stable.

But, if one component of the portfolio is not normally distributed? And if the distribution is not stationary? Or if they are not independent upon each other (they are not i.i.d)?

In all cases above, the VAR measure means nothing.

3.3 Portfolio Selection and Optimization

The Markovitch(1952) portfolio selection method is based on normal assumptions, as it uses covariances that only make sense if the variances can be summed and the components of the portfolio are i.i.d. (identically independent distributed) random variables, so their variances can be summed since a sum of i.i.d. random variables is asymptotically normal, as Lèvy has proved.

The Markovitch portfolio selection mechanism is based upon the assumption that the distribution of the assets is i.i.d. and, in consequence, the distribution of returns of the resultant portfolio is Lèvy-stable.

This is the same problem stressed in previous section with the VAR measure of financial risk.

4. Data and methodology

4.1 The data

The time-series used in this work are made available by CEPEA(2007) the "Center for Advanced Studies on Applied Economics" of ESALQ (Escola Superior de Agricultura "Luiz de Queiroz") – USP. This prices are the market's reference for spot trades and for futures and options traded in Brazilian BM&F [("Commodities Trade and Futures Exchange").

Commodity	Description	Start date	Final date	Periodicity
Soybean	60Kg sack	29-july-1997	22-jun-2007	Daily
Alcohol	Spot prices	07-july-2000	22-jun-2007	Weekly
Sugar	Spot prices w/ freight	02-may-1997	22-jun-2007	Daily
Cotton	Mean spot prices (lb – weight)	28-jun-1996	22-jun-2007	Daily
Coffee	Spot prices w/ freight	02-sep-1996	22-jun-2007	Daily

Table 1: Commodities price time-series description

4.2 Log-returns analysis

The analysis are done upon the log-returns calculated using expression below (1):

$$\mathbf{r_t = \ln p - \ln p_{t-1}} \qquad (4)$$

This is usually called log-returns, which must be normally distributed in the modern finance engineering to ensure its assumptions, as stated in previous sections (3.2).

4.3 Statistical "goodness of fit" tests performed

Some normality tests were performed using R(2006) statistical software and its nortest package (Gross (2006)). The performed normality tests were:

- Kolgomorov-Smirnov (lillie.test function in nortest package)
- Pearson Chi-square (pearson.test function in nortest package)
- Anderson-Darling (ad.test function from in package)
- Shapiro-Wilk (shapiro.test function in nortest package)
- Shapiro-Francia (sf.test function in nortest package)

For all of these tests the hipothesis are defined as follows:

- H_0: The data follow the normal distribution
- H_1: The data do not follow the normal distribution

4.3.1 The Kolgomorov-Smirnov (K.S.) test

The Kolmogorov-Smirnov test is used to decide if a sample comes from a population with a specific distribution. It is based on the empirical distribution function (ECDF). Given N ordered data points $X_{1,\ldots,}X_i$, the ECDF is defined as $S_N(X_i)$.

F_0 is the theoretical cumulative distribution of the distribution being tested which must be a continuous distribution (i.e., no discrete distributions such as the binomial or Poisson), and it must be fully specified (i.e., the location, scale, and shape parameters cannot be estimated from the data).

Test Statistic: The Kolmogorov-Smirnov test statistic is defined as

$$\mathbf{D = max \mid F_0(X_i) - S_N(X_i) \mid , i = 1,2,3,...,N} \qquad (5)$$

commonly called Kolgomorov distance.

According NIST/SEMATECH(2007), "an attractive feature of this test is that the distribution of the K-S test statistic itself does not depend on the underlying cumulative distribution function being tested. Another advantage is that it is an exact test (the chi-square goodness-of-fit test depends on an adequate sample size for the approximations to be valid)".

4.3.2 The Pearson Chi-square test

The chi-square test is used to test if a sample of data came from a population with a specific distribution.

According NIST/SEMATECH(2007), "an attractive feature of the chi-square goodness-of-fit test is that it can be applied to any univariate distribution for which you can calculate the cumulative distribution function. The chi-square goodness-of-fit test is applied to binned data (i.e., data put into classes). This is actually not a restriction since for non-binned data you can simply calculate a histogram or frequency table before generating the chi-square test.

However, the value of the chi-square test statistic are dependent on how the data is binned. Another disadvantage of the chi-square test is that it requires a sufficient sample size in order for the chi-square approximation to be valid".

The chi-square test is an alternative to the Anderson-Darling and Kolgomorov-Smirnov goodness-of-fit tests. The chi-square goodness-of-fit test can be applied to discrete distributions such as the binomial and the Poisson. The Kolgomorov-Smirnov and Anderson-Darling tests are restricted to continuous distributions.

Test Statistic: For the chi-square goodness-of-fit computation, the data are divided into k bins and the test statistic is defined as

$$\mathbf{\chi 2 = \Sigma_{i=k} [(O_i - E_i)^2 / E_i]} \qquad (6)$$

where O_i is the observed frequency for bin i and E_i is the expected frequency for bin i.

The expected frequency is calculated by the cumulative Distribution function F(Yn) for the distribution being tested.

$$\mathbf{E_i = N(F(Yu) - F(Yt))} \qquad (7)$$

4.3.3 The Anderson-Darling (A.D.) test

According NIST/SEMATECH(2007), "the Anderson-Darling test is used to test if a sample of data came from a population with a specific distribution. It is a modification of the Kolmogorov-Smirnov (K-S) test and gives more weight to the tails than does the K-S test. The K-S test is distribution free in the sense that the critical values do not depend on the specific distribution being tested. The Anderson-Darling test makes use of the specific distribution in calculating critical values. This has the advantage of allowing a more sensitive test and the disadvantage that critical values must be calculated for each distribution. Currently, tables of critical values are available for the normal, lognormal, exponential, Weibull, extreme value type I, and logistic distributions."

The Anderson-Darling test is an alternative to the chi-square and Kolmogorov-Smirnov goodness-of-fit tests, and is particularly well suited for this research on heavy tailless of the commodities price time-series.

4.3.4 The Shapiro-Wilk (S.W.) test

According NIST/SEMATECH(2007), "the Shapiro-Wilk test, proposed in 1965, calculates a W statistic that tests whether a random sample, x1, x2, ..., xn comes from (specifically) a normal distribution . Small values of W are evidence of departure from normality and percentage points for the W statistic, obtained via Monte Carlo simulations.. This test has done very well in comparison studies with other goodness of fit tests."

4.3.5 The Shapiro-Francia (S.F.) test

According with Gross(2006), "the test statistic of the Shapiro-Francia test is simply the squared correlation between the ordered sample values and the (approximated) expected ordered quantiles from the standard normal distribution. "

The Shapiro-Francia test is known to perform well. The expected ordered quantiles from the standard normal distribution are approximated by qnorm(ppoints(x, a = 3/8)), being slightly different from the approximation qnorm(ppoints(x, a = 1/2)) used for the normal quantile-quantile plot by qqnorm for sample sizes greater than 10.

4.4 Brock, Dechert, Scheinkman (BDS) statistical test of independence

The BDS test, presented by Brock, Dechert, Scheinkman, is a nonparametric test for serial independence based on the correlation integral of the scalar series, $\{x_t\}$, presented by Grassberger & Procaccia(1983):

$$C(\varepsilon) = \lim_{N\to\infty} \frac{1}{N^2} \sum_{i,j=1}^{N} \Theta(\varepsilon - ||\vec{x}(i) - \vec{x}(j)||), \quad \vec{x}(i) \in \mathbb{R}^m, \tag{8}$$

Brock, Dechert, and Scheinkman exploit the m asymptotic normality of C(epsilon) under the null hypothesis that $\{x_t\}$ is an i.i.d. process to obtain a test statistic which asymptotically converges to a unit normal. (see Kantz & Schreiber(2004) and Peters(1994))

- H_0: The data is an identically independent (i.i.d.) process
- H_1: The data is not an identically independent (i.i.d) process

5. Discussion of empirical results

The normality test results are summarized in table 1 below:

	Alchool	Sugar	Cotton	Coffee	Soybean
Kurtosis	7.122929	7.615848	12.75124	10.54829	5.604458
K.S. test (p-value)	p-value < 2.2e-16	p-value < 2.2e-16	p-value < 2.2e-16	p-value < 2.2e-16	p-value < 2.2e-16
Chi-square (p-value)	p-value = 3.82e-15	p-value < 2.2e-16	p-value < 2.2e-16	p-value < 2.2e-16	p-value < 2.2e-16
A.D. test (p-value)	p-value < 2.2e-16	p-value = NA	p-value = NA	p-value = NA	p-value < 2.2e-16
S.W. test (p-value)	p-value < 2.2e-16	p-value < 2.2e-16	p-value < 2.2e-16	p-value < 2.2e-16	p-value < 2.2e-16
S.F. test (p-value)	p-value = 3.82e-15	p-value < 2.2e-16	p-value < 2.2e-16	p-value < 2.2e-16	p-value < 2.2e-16

Table 2: Kurtosis and statistical "goodness-of-fit" tests p-value results

The null hypothesis of normality is rejected in all employed tests, as can be seen in table 2. All p-values are near zero. All tests employed allow the rejection of null hypothesis of normality. In particular, Anderson-Darling test, that give more weights to the tails than KS test, confirm the "fat-tail" behavior on the commodities log-returns time-series.

The excess kurtosis (higher than 3, the normal kurtosis) of Alcohol (7.122929) and of all other commodities price variation time-series, are above 5, what indicates that variations (volatility) are so large to be explained under the normal assumption. This large kurtosis prove the existence of "heavy tail" of the distribution of log-returns.

The BDS test rejects i.i.d. null hypothesis for soybean, cotton, coffee and sugar for all dimensions and eps values.

For alcohol, the results (table 3) of BDS test is shown below:

Eps. / [dim]	**0.0041**	**0.0173**	**0.0304**	**0.0436**	**0.0568**	**0.0699**	**0.0831**	**0.0963**	**0.1094**	**0.1226**
[2]	0.0000	0	0	0	0	0	0	0	0	0.0000
[3]	0.0000	0	0	0	0	0	0	0	0	0.0000
[4]	0.0000	0	0	0	0	0	0	0	0	0.0000
[5]	0.0000	0	0	0	0	0	0	0	0	0.0012
[6]	0.0000	0	0	0	0	0	0	0	0	0.0095
[7]	0.0000	0	0	0	0	0	0	0	0	**0.0194**
[8]	0.0000	0	0	0	0	0	0	0	0	**0.0106**
[9]	**0.5679**	0	0	0	0	0	0	0	0	0.0066
[10]	**0.6662**	0	0	0	0	0	0	0	0	0.0034

Table 3: BDS test results for alcohol time-series

As can be seen in the table 3, the i.i.d. null hypothesis cannot be rejected for embedding dimensions greater than 9. With the e.p.s. > 0.1226 and dimensions 7 and 8 the null hypothesis cannot either be rejected in 99% significance level for the alcohol price log-returns time-series.

This results seem to be spurious and probably result from the presence of "false-neighbors" that distorted the correlation integral upon this test is based on (see Kantz and Schreiber(2004)).

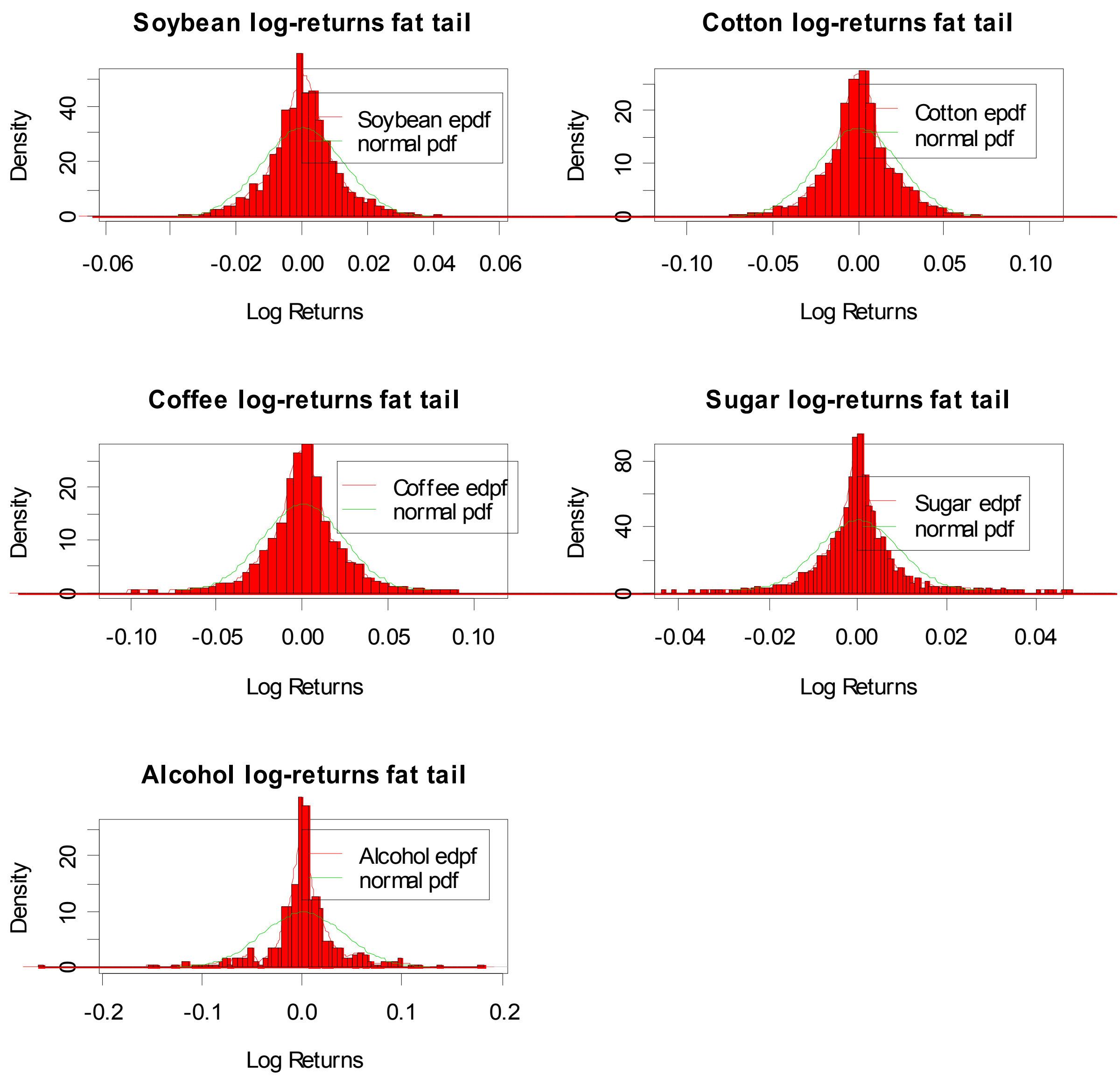

Figure 1: Empirical pdf of price variations compared with normal pdf

Figure 1 shows the histogram of log-returns for all anallised time-series compared with the normal curve with the same mean and sample variance. It is clear the long tail, as the empirical probability distribution (e.p.d.f.) of commodities time-series exhibit variation far from three standard deviations (99,9%), what is unacceptable under normal assumption.

This results allow the rejection of normality and i.i.d. null hypothesis in the analyzed time-series.

6. Conclusions and further researches

The modern finance engineering assumptions were shown to be Gaussian dependent. The stylized facts of finance time-series, specially the presence of long tails, and its impact on the finance engineering tools were shown.

Formal statistical tests of normality and independence (i.i.d.) were employed and allow to reject the null hypothesis of normality and independence.

The options price model by Black-Scholes, the Capital Asset Pricing Model, the risk models used by authorities to regulate banking and trading (Basle agreement); all of them presumes normality. And, according to results found in this and in many others researches, this assumptions are clearly wrong. As consequence, options are mis-precified, assets are mis-precified, risks are mis-evaluated and banking and trade activities are mis-regulated.

The Brazilian commodities price variations studied were shown to be non-Gaussian nor independent. This means that usual tools used to precify assets like options or futures, in order to do hedging, for example, need to account for this behavior; and they do not. As a consequence, prices and risks are mis-evaluated, what has the potential to induce bad decisions.

As a result, the finance engineering tools need to be changed in order address the non-Gaussian behavior of time-series; and many models have been proposed to model this financial time-series behavior, like GARCH, ARIMA. FARIMA, ARSV and so on. Maldelbrot proposes the Fractional Time Brownian Motion as a model of financial time-series behavior; but the best model is a open question of research (for more references in these models, see Elder & Serletis(2007), Mandelbrot(1997) and MacNeil & Frei(1998)).

In this work this models are not tested, although this can be done in the future.

7. References

[1] Bradley B.O. and Taqqu M.S. (2001), Financial Risk and Heavy Tails, Preprint, Boston University. http://citeseer.ist.psu.edu/bradley02financial.html

[2] Davis, M. and Etheridge,A. (2006), Louis Bachelier's Theory of Speculation: The Origins of Modern Finance, Princeton University Press.

[3] Elder J. and Serletis, A. (2007), Long memory in energy futures prices. Review of Financial Economics , doi: 10.1016/j.rfe.2006.10.002.

[4] Embrechts, P. et al. (1997). Modelling Extreme Events for Insurance and Finance. Annals of Applied Probability 6:1191--1210.Springer-Verlag, Berlin.

[5] CEPEA (2007)– Center for Advanced Studies on Applied Economics – ESALQ / USP, accessed 7-july-2007, online: http://cepea.esalq.usp.br/english

[6] Giesecke, K et al. (2005) , Measuring the Risk of Extreme Events

[7] Glasserman,P. (1962) Monte Carlo Methods in Financial Engineering, Springer

[8] Gross, J. (2006). Nortest: Tests for Normality. R package version 1.0.

[9] Hyung N. and DeVries (2007), C. G., Portfolio selection with heavy tails, Journal of empirical finance 14, p. 383-400, 14, doi:10.1016/j.jempfin.2006.06.004,.

[10]Kim K. and Yoon S.M. (2004), Multifractal features of financial markets. Physica A 344, p. 272-278, doi:10.1016/j.physa.2004.06.131

[11]Kantz, H. and Schreiber (2004), T., Nonlinear Tome Series Analysis, 2ed., Cambridge University Press, UK

[12]Laskin, N. (2000), Fractional market dynamics, Physica A 287, 482-492, Elsevier

[13]Maldelbrot, B.B. (1963) The variation of certain speculative prices. Journal of Business, 36:392—417.

[14]Maldelbrot, B. (1963b), New methods i n statistical economics, The Journal of Political Economy, n. 71, p. 421-440.

[15]Maldelbrot,B. (1997) Fractals and Scaling in Finance, Springer

[16]Mandelbrot, B. and Hudson R.L. (2004) The (Mis)Behavior of Markets, Basic Books

[17]McNeil, A.J. and Frey, R. (1998). Estimation of tail-related risk measures for heteroscedastic financial time series: an extreme value approach. Preprint, Dept. of Mathematics, ETH Zurich, Switzerland. http://citeseer.ist.psu.edu/mcneil98estimation.html

[18]NIST/SEMATECH (2007) e-Handbook of Statistical Methods, http://www.itl.nist.gov/div898/handbook/, 7-july-2007.

[19]Peters, E.E. (1994), Fractal Market Analysis: Applying Chaos Theory to Investment and Economics, Wiley & Sons, New York.

[20]Plerou V. et al. I (2000), Econophysics: financial time series from a statistical physics point of view, Physica A, 279, p. 443-456, Elsevier

[21]Grassberger P. and Procaccia I. (1983). "Measuring the strangeness of strange attractors". Physica 9D: 189-208

[22]R Development Core Team (2006). R: A language and environment for statistical computing. R Foundation for Statistical Computing, Vienna, Austria. ISBN 3-900051-07-0, URL http://www.R-project.org.

[23]Siegel S and Castellan N. J. (1988) Nonparametric Statistics for the Behavioral Sciences, McGraw-Hill, New York

[24]Sun,W., et al. (2007) , Fractals or I.I.D.: Evidence of long-range dependence and heavy tailedness from modeling German equity market returns, Journal of Economics and Business, doi:10.1016/j.jeconbus.2007.02.001

[25]Markowitz, H. (1952). Portfolio Selection. The Journal of Finance, vol 7, n 1, pp 77-91, mar 1952